STT 200 1-30-09a

NOTE: IN LECTURE 1-28-09

7(c) $s \sim 4.51 \times 10^{-4}$ MAKE CORRECTION (ITS BEEN DONE IN THE POST)

~~TODAY~~: ~~FINISH HANDOUT 1-26-09~~ AND CONTINUE GENERAL REVIEW.

#8. LIST x s_x

CLAIM 1. $s_{(x+6)} = \sqrt{\dfrac{(x_1+\cancel{6}-(\bar{x}+\cancel{6}))^2 + \cdots + (x_n+6-(\bar{x}+6))^2}{n-1}}$

$= s_x$

$\overline{x+6} = \dfrac{x_1+6+x_2+6 \cdots + x_n+6}{n}$

ADD 6 TO EVERY SCORE ON LIST THEN AVG.

$= \dfrac{x_1+\cdots+x_n}{n} + \dfrac{\cancel{n}6}{\cancel{n}}$

$= \bar{x}+6$

RULE: $s_{x+c} = s_x$

MAY BE NEGATIVE!

CLAIM 2. $s_{cX} = \sqrt{\dfrac{(cx_1 - c\bar{x})^2 + \cdots + (cx_n - \overline{cx})^2}{n-1}}$

cx_1

cx_2

$\vdots$

cx_n

$= \sqrt{c^2 \dfrac{(x_1-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n-1}}$

$= |c|\, s_X$

GENERAL RULE IS

$s_{cX} = |c|\, s_X$

$\overline{cX} = \dfrac{cx_1 + cx_2 + \cdots + cx_n}{n}$

MULTIPLY EVERYONE'S \$ BY c THEN AVERAGE

$= \dfrac{c(x_1 + \cdots + x_n)}{n}$

$= c\bar{x}$

SAME IDEA DONE WITH TABLE

LIST $\{0, 4, 8\}$ $n = 3$

x	$(x-\bar{x})^2$	$5x$	$(5x-20)^2$
0	$(0-4)^2 = 16$	0	$(0-20)^2 = 400$
4	$(4-4)^2 = 0$	20	$(20-20)^2 = 0$
8	$(8-4)^2 = 16$	40	$(40-20)^2 = 400$
TOT 12	32	60	800
AVG 4	$\boxed{32/(3-1)} = 16$	20	$\boxed{800/(3-1) = 400}$

$s_x = \sqrt{16} = 4$

$s_{5x} = \sqrt{400} = 20$

$= |5| s_x$

$= 5 \cdot 4$

$(c = 5)$

$\overline{5x} = 20$

$= 5\bar{x}$

$= 5(4)$

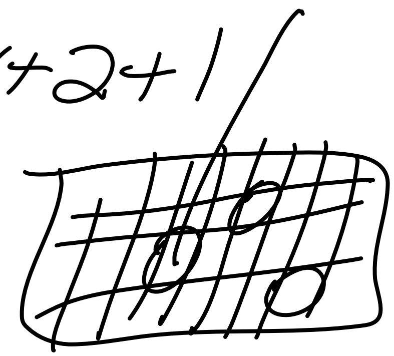
PER #8 ESTIMATING CROWD SIZE.

AREA DIVIDED INTO 1056 PATCHES.

PEOPLE # $1056\,\mu$ $\quad \mu$ = AVG # OF PEOPLE PER PATCH.

4 $\qquad$ 2 $\qquad$ 1 $\qquad$ TOTAL = 4+2+1

$$\frac{4+2+1}{3} \text{ AVG \# PER PATCH}$$



$$\frac{4+2+1}{\cancel{3}}\,\cancel{3} = \text{TOTAL.}$$

WE WILL ESTIMATE μ.

POPULATION ≡ PATCHES.

SCORE x OF A PATCH ≡ # PEOPLE IN IT

SUPPOSE SAMPLE 100 PATCHES FINDING

$\overline{X} = 62.55$

x_1 = # PEOPLE IN SAMPLE PATCH 1

$\vdots$

x_{100} = ... " PATCH 100

AVG # OF PEOPLE PER PATCH IN 100 SAMPLE PATCHES

s = SAMPLE S.D FOR X-LIST.

$= 23.49$

$$\sqrt{\frac{1056-100}{1056-1}}$$

95% CI FOR μ IS $62.55 \pm 1.96 \dfrac{23.49}{\sqrt{100}}$

EST OF TOTAL 1056μ IS $1056\overline{X}$

$$1056(62.55) \pm 1.96 \cdot 1056 \frac{23.49}{\sqrt{100}}$$

95% CI FOR TOTAL CROWD SIZE

STT 200 1-30-09

1% OF TOTAL CROWD IS 1056 $\bar{X}$

WE DESIRE ESTIMATE OF TOTAL CROWD SIZE

BECAUSE TOTAL CROWD = 1056 μ

REGION IN WHICH THIS CROWD IS ASSEMBLED

NUMBERS 1056 PATCHES

μ = AVE # PEOPLE PER PATCH IN TOTAL CROWD

X = # PEOPLE

RELY UPON FACT $1.96 \sigma_x \sqrt{\frac{1056-100}{1}}$

THAT TOTAL CROWD SIZE

AND OUR ESTIMATE IS 1056 $\bar{X}$

$\bar{X}$ ESTIMATE FOR 1056 $\bar{X}$ IS

POPULATION μ = AVE # PEOPLE PER PATCH

WE CAN RANDOMLY SAMPLE n = 100 PATCHES

GET SCORES $x_1, x_2, \ldots$ X_1 = # PEOPLE IN SAMPLE PATCH 1

$1.96\,\sigma_x \sqrt{\frac{1056-100}{1056}}$

POPULATION ARE 1056 PATCHES.

8a. OUR SAMPLE $\bar{x} = \frac{x_1 + \cdots + x_{100}}{100} = 62.55$ WE'RE TOLD THIS

POINT EST OF TOTAL CROWD IS THEN

$1056(62.55)$

COULD TAKE NEW SCORES

$y_i = 1056\, x_i$

$\bar{y} = 1056\, \bar{x}$

8b.

SO EMOE FOR $\bar{y}$

$$= 1.96 \frac{\sigma_y}{\sqrt{n}} \sqrt{\frac{1056-100}{1056-1}}$$

$$= 1.96 \frac{\sigma_{1056x}}{\sqrt{100}} \sqrt{\frac{1056-100}{1056-1}}$$

$$= 1.96 \frac{1056\, \sigma_x}{\sqrt{100}} \sqrt{\frac{1056-100}{1056-1}}$$

8c. 95% CI FOR TOTAL CROWD SIZE 1056μ

TARGET 1056μ

(PT EST $1056\bar{x}$) $= 1056(62.55)$

E MOE OF $1056\bar{x}$ IS $1.96 \dfrac{1056\sigma_x}{\sqrt{n}} \sqrt{\dfrac{1056-100}{1056-1}}$

GIVEN $\sigma_x = 23.49$

SO 95% CI:

$$1056(62.55) \pm 1.96 \; 1056 \; \frac{23.49}{\sqrt{100}} \sqrt{\frac{1056-100}{1056-1}}$$

? SUPPOSED TO HAVE $\sigma_{1056x} = 1056\sigma_x$

PROPERTIES OF s_x.

$x \to x+6$

$$\boxed{s_{x+\cancel{6}} = s_x}$$

x = YOUR \$

GIVE \$6 TO EACH PERSON

$x \to x+6$

$\bar{x}$ vs $\overline{x+6}$

$$\overline{x+6} = \frac{(x_1+6)+(x_2+6)+\cdots+(x_n+6)}{n}$$

$$= \frac{x_1+\cdots+x_n+n6}{n} = \bar{x} + \frac{\cancel{n}6}{\cancel{n}}$$

so $\overline{x+6} = \bar{x}+6$

WHAT ABOUT s?

$$s_{x+6} = \sqrt{\frac{((x_1+\cancel{6})-(\bar{x}+\cancel{6}))^2+\cdots+((x_n+\cancel{6})-(\bar{x}+\cancel{6}))^2}{n-1}} = s_x$$

WHAT ABOUT s_{2x} ?

$$\overline{2x} = \frac{2x_1 + 2x_2 + \cdots + 2x_n}{n}$$

$$= 2\bar{x}$$

$$s_{2x} = \sqrt{\frac{(2x_1 - 2\bar{x})^2 + \cdots + (2x_n - 2\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{4(x_1 - \bar{x})^2 + \cdots + 4(x_n - \bar{x})^2}{n-1}}$$

$$= 2\, s_x$$

$$s_{cx} = |c|\, s_x$$

$$\sqrt{(-4)^2}$$

$$= 4$$

TODAY - WE LEARNED THAT:

$$y = ax+b \qquad \bar{y} = a\bar{x}+b$$

$$\sigma_y = \sigma_{ax+\cancel{b}}$$

95% CI for μ_x

is $\bar{x} \pm 1.96 \frac{\sigma_x}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

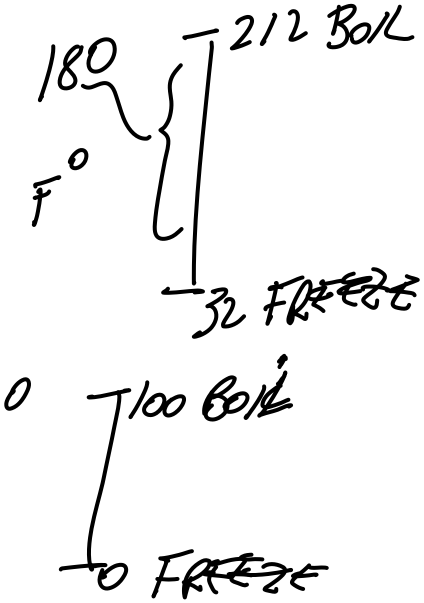
95% CI for $\mu_y = \mu_{ax+b}$

is $(a\bar{x}+b) \pm 1.96 \frac{\overset{|a|\sigma_x}{\sigma_{ax+\cancel{b}}}}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

$= (a\bar{x}+b) \pm 1.96\, |a| (\sigma_x / \sqrt{n}) \sqrt{\frac{N-n}{N-1}}$

e.g. SUPPOSE WE'VE A 95% CI FOR μ_{F°

$$F^\circ = \underbrace{\frac{180}{100}}_{a} C^\circ + \underbrace{32}_{b}$$



$$F = aC + b \equiv C = \frac{F-b}{a}$$

SO GIVEN 95% CI for μ_{F°

IS $\bar{x}_{F^\circ} \pm 1.96 \frac{\sigma_{F^\circ}}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

$C^\circ = \frac{5}{9}F^\circ -$

IN C°

$$\bar{x}_{C^\circ} \pm 1.96 \frac{\sigma_{C^\circ}}{\sqrt{n}} \sqrt{\frac{N-1}{N-1}}$$

$\sigma_{C^\circ} = \sigma_{5/9 F^\circ + \text{[illegible]}}$

$$(9/5\, \bar{x}_{F^\circ} + 32) \pm 1.96 (5/9)\, \sigma_{F^\circ} / \sqrt{n}\, (\text{fc}) \quad = (5/9)\, \sigma_{F^\circ}$$